NAME _KEY_ _____ CLASS _____ DATE _____

Solve each of the following:

1. $\dfrac{x}{6} = \dfrac{3}{21}$ _____

2. $\dfrac{51}{90} = \dfrac{30}{x}$ _____

3. $\dfrac{0.7071}{45} = \dfrac{0.7660}{x}$ _____

4. $\dfrac{0.5}{30} = \dfrac{x}{45}$ _____

5. $\dfrac{300}{x} = 0.0152$ _____
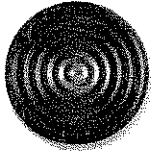
Lesson Quiz
14.1 The Law of Sines

1. Find the area of the triangle shown at right to the nearest tenth of a square centimeter.
   _____ 213.8 _____

   20 cm / 24 cm / 72° / 45°

2. Find x and y in the triangle shown at right to the nearest tenth of a meter.
   _____ $x \approx 12.2$ $y \approx 28.0$ _____

   87° / x / y / 24° / 30 m
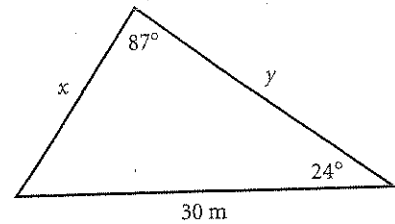
3. Find x in the triangle shown at right to the nearest tenth of a degree.
   _____ 44.2° _____

   95° / 35 cm / x / 50 cm

4. In a triangle, $a = 86$, $b = 63$, and $m\angle B = 47°$.
   a. Does the given information determine a triangle? _____ YES _____
   b. If it does, find all possible values for $m\angle A$ to the nearest tenth of a degree. _____ $\angle A \approx 86.7°$ or $93.3°$ _____

Quick Warm-Up: Assessing Prior Knowledge
14.2 The Law of Cosines

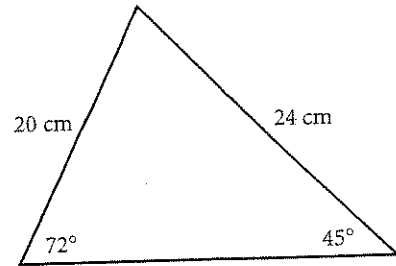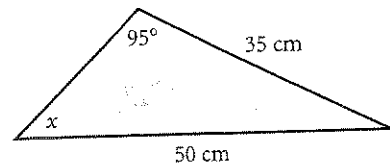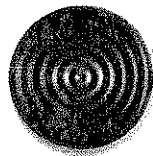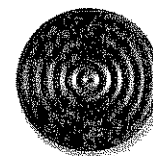In each exercise, find the unknown quantity. Round angles to the nearest degree and lengths to the nearest tenth.

1. $\theta = \sin^{-1} 0.39$ _____

2. $\cos \theta = 0.96$ _____

3. $8^2 + 9^2 - 2(8)(9)\cos \theta = 3^2$ _____

4. In $\triangle ABC$, $a = 27$, $b = 18$, and $m\angle C = 110°$.
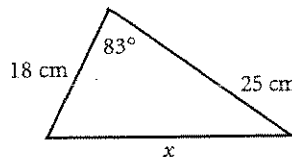   Find the area of the triangle. _____

5. For $\triangle XYZ$, $x = 10$, $y = 12$, and $X = 44°$.
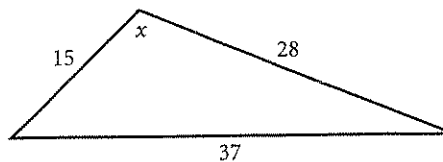   Solve the triangle. _____

Lesson Quiz
14.2 The Law of Cosines

1. Find x in the triangle shown at right to the nearest tenth of a centimeter.
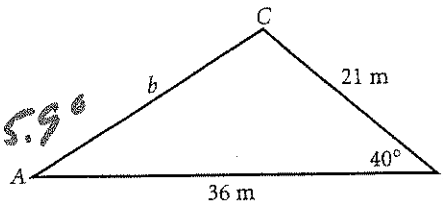
   _29. 0°_

18 cm   83°   25 cm
x

2. Find x in the triangle shown at right to the nearest tenth of a degree.

   _115.4°_

15   x   28
37

3. Solve $\triangle ABC$. Find b to the nearest tenth of a meter and $m\angle A$ and $m\angle C$ to the nearest tenth of a degree.

   _b ≈ 24.1   m∠A ≈ 34.1°   ∠C ≈ 105.9°_

C
b   21 m
A   36 m   40°   B

4. A surveyor is measuring the distance across a pond, represented by YZ in the diagram at right. He finds that point X is 150 yards from point Y and 200 yards from point Z. If the measure of the angle formed at point X is 42°, what is the distance across the pond?
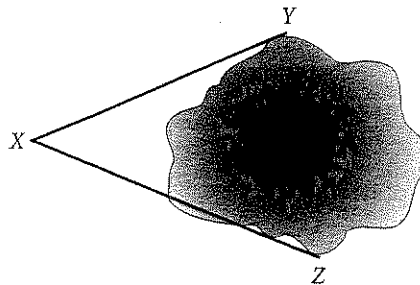
   _133. 8_

Y
X
Z

Algebra 2

Quick Warm-Up: Assessing Prior Knowledge

14.3 Fundamental Trigonometric Identities

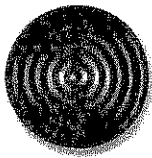Given the information below, solve for every possible $\triangle ABC$.
Round angle measures to the nearest degree and lengths to the
nearest tenth of a unit.

1. $a = 3, b = 4, c = 5$ _____

2. $A = 45°, B = 30°, c = 10$ _____

3. $B = 120°, a = 12, c = 10$ _____

4. $A = 45°, a = 10, b = 12$ _____

5. $B = 30°, a = 12, b = 4$ _____

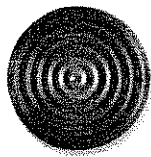# Lesson Quiz

14.3 Fundamental Trigonometric Identities

Write each expression in terms of a single trigonometric function.

1. $(\cot \theta)(\sec \theta)$

   csc θ

2. $\dfrac{\sec \theta}{\csc \theta}$

   tan θ

3. $(\csc \theta + 1)(\csc \theta - 1)$

   cot² θ

4. $(\tan^2 \theta + 1)(\cos \theta)$

   sec θ

5. $\dfrac{\sin^2 \theta}{1 - \sin^2 \theta}$

   tan² θ

6. $\dfrac{\cos^2 \theta + \sin^2 \theta}{(\sec \theta)(\cot \theta)}$

   sin θ

7. $\dfrac{\sec^2 \theta - 1}{\tan \theta}$

   tan θ

8. $\dfrac{\cos \theta}{(\sin \theta)(\cot \theta)}$

   1

Write each expression in terms of $\tan \theta$.

9. $\dfrac{(\sec \theta)(\sin \theta)}{\tan \theta + \cot \theta}$

   $\dfrac{\tan^2 \theta}{\tan^2 \theta + 1}$

10. $\dfrac{1 - \cos^2 \theta}{\cos^2 \theta}$

    tan² θ

11. $\dfrac{\sin \theta}{\cos \theta} + \dfrac{\cos \theta}{\sin \theta}$

    $\tan \theta + \dfrac{1}{\tan \theta}$

12. $\csc^2 \theta - 1$

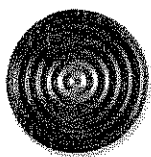   *$\dfrac{1}{\tan^2 \theta}$*

Algebra 2

Quick Warm-Up: Assessing Prior Knowledge
14.4 Sum and Difference Identities

Find the distance between each pairs of points below. Give
approximate values rounded to the nearest tenth.

1. $(4, 1)$ and $(4, 0)$ _____

2. $(3, 2)$ and $(-2, -5)$ _____

3. $(-5, 6)$ and $(5, 1)$ _____

4. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ _____

5. $(\cos 30°, \sin 30°)$ and $(\cos 45°, \sin 45°)$ _____

Lesson Quiz
14.4 Sum and Difference Identities

Find the exact value of each expression.

1. $\sin\left(\frac{5\pi}{3} - \frac{\pi}{6}\right)$ $\quad -1$

2. $\cos\left(\frac{3\pi}{2} + \frac{3\pi}{4}\right)$ $\quad \frac{\sqrt{2}}{2}$

3. $\sin(-210°)$ $\quad \frac{1}{3}$

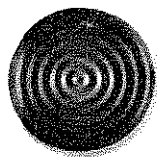4. $\cos 75°$ $\quad \frac{\sqrt{6} - \sqrt{2}}{4}$

Prove each identity.

5. $\sin(\theta - 270°) = \cos \theta$

6. $\cos(\pi + x) = -\cos x$

7. Find the rotation matrix for a rotation of $120°$ about the origin.
Then find the image of the point $(3, 4)$ after this rotation. Give
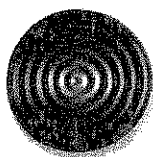coordinates rounded to the nearest hundredth.

Quick Warm-Up: Assessing Prior Knowledge
14.5 Double-Angle and Half-Angle Identities

Find the exact value of each expression:

1. $\sin 15°$ _____

2. $\cos 15°$ _____

3. $\sin 255°$ _____

4. $\cos 255°$ _____

5. $\tan(-255°)$ _____

Lesson Quiz
14.5 Double-Angle and Half-Angle Identities

Use the information given to find the exact value of $\sin 2\theta$ and $\cos 2\theta$.

1. $0° \leq \theta \leq 90°$; $\sin \theta = \frac{4}{5}$

$\sin 2\theta = \frac{24}{25}$ $\cos 2\theta = \frac{-7}{25}$

2. $0° \leq \theta \leq 90°$; $\sin \theta = \frac{2}{\sqrt{5}}$

$\sin 2\theta = \frac{4}{5}$ $\cos 2\theta = -\frac{3}{5}$

3. $270° \leq \theta \leq 360°$; $\cos \theta = \frac{3}{5}$

$\sin 2\theta = -\frac{24}{25}$ $\cos 2\theta = -\frac{7}{25}$

4. $180° \leq \theta \leq 270°$; $\cos \theta = -\frac{5}{13}$

$\sin 2\theta = \frac{120}{169}$ $\cos 2\theta = \frac{-119}{169}$

Use the information given to find the exact value of $\sin \frac{\theta}{2}$ and $\cos \frac{\theta}{2}$.
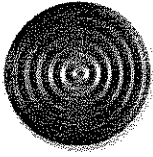
5. $270° \leq \theta \leq 360°$; $\sin \theta = -\frac{12}{13}$

$\sin \frac{\theta}{2} = \frac{2\sqrt{13}}{13}$ $\cos \frac{\theta}{2} = -\frac{3\sqrt{13}}{13}$

6. $270° \leq \theta \leq 360°$; $\sin \theta = -\frac{4}{5}$

$\sin \frac{\theta}{2} = \frac{\sqrt{5}}{5}$ $\cos \frac{\theta}{2} = -\frac{2\sqrt{5}}{5}$

7. $180° \leq \theta \leq 360°$; $\cos \theta = \frac{7}{25}$

$\sin \frac{\theta}{2} = \frac{3}{5}$ $\cos \frac{\theta}{2} = -\frac{4}{5}$

8. $180° \leq \theta \leq 360°$; $\cos \theta = -\frac{7}{25}$

$\sin \frac{\theta}{2} = \frac{4}{5}$ $\cos \frac{\theta}{2} = -\frac{3}{5}$

Write each expression in terms of a single trigonometric function of θ.

9. $\frac{2 \sin \theta}{\sin(2\theta)}$

$\frac{1}{\cos \theta} = \sec \theta$

10. $\cos(2\theta) + \sin^2 \theta$

$\cos^2 \theta$

Quick Warm-Up: Assessing Prior Knowledge
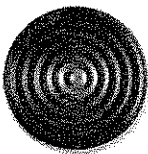## *14.6* Solving Trigonometric Equations

Solve for x.

1. $2x^2 + x - 1 = 0$ _____

2. $x^2 - 2x - 3 = 0$ _____

Solve for $0° \leq \theta \leq 360°$.

3. $\sin \theta = -1$ _____

4. $\cos \theta = -\dfrac{\sqrt{3}}{2}$ _____

5. $\sin \theta = \dfrac{\sqrt{6} - \sqrt{2}}{4}$ _____

Lesson Quiz
14.6 Solving Trigonometric Equations

Find the exact solutions of each equation for $0° \leq \theta \leq 360°$.

1. $6 \cos \theta + 4 = 1$
    $120° \, \& \, 240°$

2. $5 \sin \theta - \sqrt{2} = 3 \sin \theta$
    $45°, 135°$

3. $4 \sin \theta + 1 = 3$
    $30°, 150°$

4. $3 \cos \theta + 2 = -\cos^2 \theta$
    $180°$

Find the exact solutions of each equation for $0 \leq x \leq 2\pi$.

5. $\sin^2 x + 3 \sin x - 4 = 0$ $\dfrac{\pi}{2}$

6. $3 - 3 \cos x = 2 \sin^2 x$
    $0, 2\pi, \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

7. $2 \sin x = -\sqrt{3}$ $\dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

8. $2 \sin x = 4 \sin x + 2$ $\dfrac{3\pi}{2}$

Find all solutions of each equation.
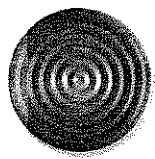
9. $5 \cos \theta + 2 = 2 - 3 \cos \theta$
    $90° + 360°n, 270° + 360°n$

10. $\sin 2\theta = 4 \sin \theta$
    $180°n$

11. $3 \sin^2 \theta - \cos^2 \theta = 0$
    $30° + 360°n, 150° + 360°n$

12. $\cos 2\theta + \sin \theta = 1$
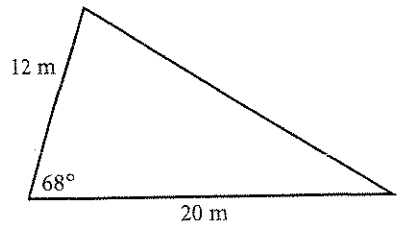 $180°n, 30° + 360°n, 150° + 360°n$
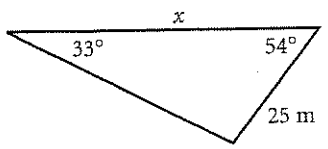
Chapter Assessment

Chapter 14, Form A, page 1

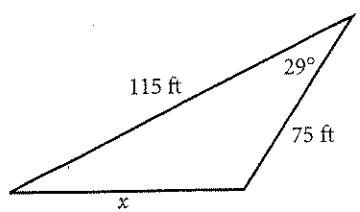Write the letter that best answers the question or completes the statement.

___B___ 1. Find the area of the triangle shown at right.

   a. 44.95 square meters

   b. 111.26 square meters

   c. 120 square meters

   d. 222.52 square meters

12 m
68°
20 m

___D___ 2. Find the value of x in the triangle shown at right.

   a. 13.6 meters   b. 24.8 meters

   c. 37.1 meters   d. 45.8 meters

x
33°   54°
25 m

___D___ 3. Find the value of x in the triangle shown at right.

   a. 106.3 feet   b. 102.4 feet

   c. 83.5 feet   d. 61.3 feet

29°
115 ft
75 ft
x

___C___ 4. Find the smallest angle in the triangle shown at right.

   a. 24.6°   b. 31.2°

   c. 40.3°   d. 63.5°

68 cm   94 cm
102 cm

___A___ 5. Find the value of x in the triangle shown at right.

   a. 50.1°   b. 55.3°

   c. 59.7°   d. 64.4°

74
35° x
57

___A___ 6. In $\triangle ABC$, $b = 42.9$, $c = 39.5$, and $m\angle B = 80°$. Find $m\angle C$.

   a. 65.1°   b. 114.9°

   c. 65.1° or 114.9°   d. none of these

___C___ 7. Which of the following is a simpler form of $\dfrac{(\csc\theta)(\tan\theta)}{(\cot\theta)(\sec\theta)}$?

   a. $\cos\theta$ b. $\sin\theta$ c. $\tan\theta$ d. $\sec\theta$

Chapter Assessment

Chapter 14, Form A, page 2

_B_ 8. Which of the following is a simpler form of $\frac{\sec^2 \theta}{(\sec \theta + 1)(\sec \theta - 1)}$?

     a. $\cot^2 \theta$ b. $\csc^2 \theta$ c. $\cos^2 \theta$ d. $\tan^2 \theta$

_B_ 9. Which of the following is a simpler form of $\frac{\sin^2 \theta}{1 - \cos \theta} - 1$?

     a. $\sec \theta$ b. $\cos \theta$ c. $\csc \theta$ d. $\sin \theta$

_D_ 10. Which of the following is a simpler form of $\cos(\theta + 90°)$?

     a. $\tan \theta$ b. $-\tan \theta$ c. $\sin \theta$ d. $-\sin \theta$

_D_ 11. Which of the following is a simpler form of $\sin(3\pi + x)$?

     a. $\cos x$ b. $-\cos x$ c. $\sin x$ d. $-\sin x$

_C_ 12. Which of the following is a simpler form of $(\sec^2 \theta)(\sin 2\theta)$?

     a. $2 \sin \theta$ b. $2 \csc \theta$ c. $2 \tan \theta$ d. $2 \cot \theta$

_D_ 13. Which of the following is a simpler form of $\sin^2 \theta + \cos^2 \theta + \cos 2\theta$?

     a. $2 \tan^2 \theta$ b. $2 \sin^2 \theta$ c. $2 \csc^2 \theta$ d. $2 \cos^2 \theta$

_A_ 14. Find the exact value of $\sin\left(\frac{\pi}{6} - \frac{2\pi}{3}\right)$.

     a. -1 b. $-\frac{\sqrt{3}}{3}$ c. $\frac{1}{2}$ d. $\frac{\sqrt{3}}{3}$

_D_ 15. If $\cos \theta = \frac{1}{8}$ and $270° \leq \theta \leq 360°$, find $\cos\left(\frac{\theta}{2}\right)$.

     a. $\frac{1}{2}$ b. $\frac{3}{4}$ c. $-\frac{1}{2}$ d. $-\frac{3}{4}$

_A_ 16. Solve $7 \sin \theta = 4.7 + 2 \sin \theta$ for $0° \leq \theta \leq 360°$.

     a. 70° and 110°   b. 70° and 290°   c. 31.5° and 148.5°   d. 31.5° and 328.5°

_B_ 17. Solve $8 \cos x + 2\sqrt{3} = 4 \cos x$ for $0 \leq x \leq 2\pi$.

     a. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ b. $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$ c. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ d. $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$

_D_ 18. Which equation would have to be solved in order to solve $3 \sin^2 x + 5 \sin x = 2$?

     a. $\sin x = 3$ and $\sin x = -2$ b. $\sin x = 3$ and $\sin x = -\frac{1}{2}$

     c. $\sin x = -3$ and $\sin x = 2$ d. $\sin x = \frac{1}{3}$ and $\sin x = -2$

_C_ 19. If an isosceles triangle has a base of length 36 and its vertex angle measures 48°, find the perimeter of the triangle.

     a. 75.4   b. 116.9   c. 124.5   d. 192

Chapter Assessment

Chapter 14, Form A, page 1

Write the letter that best answers the question or completes the statement.

___B___ 1. Find the area of the triangle shown at right.

    a. 44.95 square meters

    b. 111.26 square meters

    c. 120 square meters

    d. 222.52 square meters
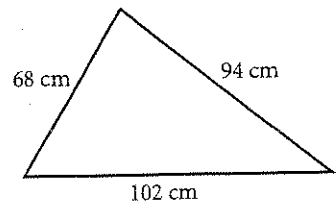
12 m, 68°, 20 m

___D___ 2. Find the value of x in the triangle shown at right.

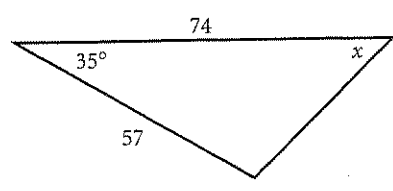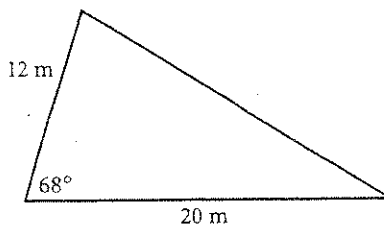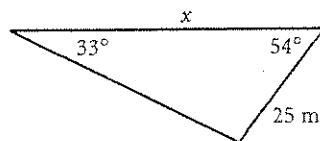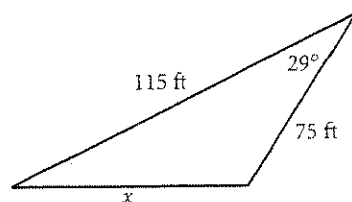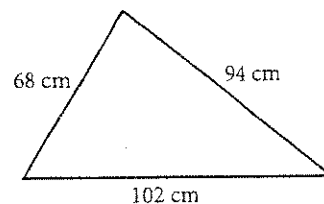   a. 13.6 meters   b. 24.8 meters

    c. 37.1 meters   d. 45.8 meters
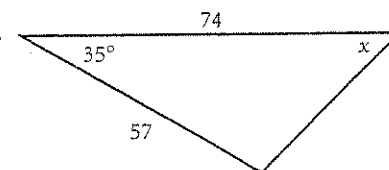
x, 33°, 54°, 25 m

___D___ 3. Find the value of x in the triangle shown at right.

    a. 106.3 feet   b. 102.4 feet

    c. 83.5 feet   d. 61.3 feet

115 ft, 29°, 75 ft, x

___C___ 4. Find the smallest angle in the triangle shown at right.

    a. 24.6°   b. 31.2°

    c. 40.3°   d. 63.5°

68 cm, 94 cm, 102 cm

___A___ 5. Find the value of x in the triangle shown at right.

    a. 50.1°   b. 55.3°

    c. 59.7°   d. 64.4°

74, 35°, x, 57

___A___ 6. In $\triangle ABC$, $b = 42.9$, $c = 39.5$, and $m\angle B = 80°$. Find $m\angle C$.

    a. 65.1°   b. 114.9°

    c. 65.1° or 114.9°   d. none of these

___C___ 7. Which of the following is a simpler form of $\dfrac{(\csc\theta)(\tan\theta)}{(\cot\theta)(\sec\theta)}$?

    a. $\cos\theta$ b. $\sin\theta$ c. $\tan\theta$ d. $\sec\theta$

_B_ 8. Which of the following is a simpler form of $\dfrac{\sec^2 \theta}{(\sec \theta + 1)(\sec \theta - 1)}$?

    a. $\cot^2 \theta$ b. $\csc^2 \theta$ c. $\cos^2 \theta$ d. $\tan^2 \theta$

_B_ 9. Which of the following is a simpler form of $\dfrac{\sin^2 \theta}{1 - \cos \theta} - 1$?

    a. $\sec \theta$ b. $\cos \theta$ c. $\csc \theta$ d. $\sin \theta$

_D_ 10. Which of the following is a simpler form of $\cos(\theta + 90°)$?

    a. $\tan \theta$ b. $-\tan \theta$ c. $\sin \theta$ d. $-\sin \theta$

_D_ 11. Which of the following is a simpler form of $\sin(3\pi + x)$?

    a. $\cos x$ b. $-\cos x$ c. $\sin x$ d. $-\sin x$

_C_ 12. Which of the following is a simpler form of $(\sec^2 \theta)(\sin 2\theta)$?

    a. $2 \sin \theta$ b. $2 \csc \theta$ c. $2 \tan \theta$ d. $2 \cot \theta$

_D_ 13. Which of the following is a simpler form of $\sin^2 \theta + \cos^2 \theta + \cos 2\theta$?

    a. $2 \tan^2 \theta$ b. $2 \sin^2 \theta$ c. $2 \csc^2 \theta$ d. $2 \cos^2 \theta$

_A_ 14. Find the exact value of $\sin\left(\dfrac{\pi}{6} - \dfrac{2\pi}{3}\right)$.

    a. -1 b. $-\dfrac{\sqrt{3}}{3}$ c. $\dfrac{1}{2}$ d. $\dfrac{\sqrt{3}}{3}$

_D_ 15. If $\cos \theta = \dfrac{1}{8}$ and $270° \leq \theta \leq 360°$, find $\cos\left(\dfrac{\theta}{2}\right)$.

    a. $\dfrac{1}{2}$ b. $\dfrac{3}{4}$ c. $-\dfrac{1}{2}$ d. $-\dfrac{3}{4}$

_A_ 16. Solve $7 \sin \theta = 4.7 + 2 \sin \theta$ for $0° \leq \theta \leq 360°$.

    a. 70° and 110°   b. 70° and 290°   c. 31.5° and 148.5°   d. 31.5° and 328.5°

_B_ 17. Solve $8 \cos x + 2\sqrt{3} = 4 \cos x$ for $0 \leq x \leq 2\pi$.

    a. $\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$ b. $\dfrac{5\pi}{6}$ and $\dfrac{7\pi}{6}$ c. $\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$ d. $\dfrac{3\pi}{4}$ and $\dfrac{5\pi}{4}$

_D_ 18. Which equation would have to be solved in order to solve $3 \sin^2 x + 5 \sin x = 2$?

    a. $\sin x = 3$ and $\sin x = -2$ b. $\sin x = 3$ and $\sin x = -\dfrac{1}{2}$

    c. $\sin x = -3$ and $\sin x = 2$ d. $\sin x = \dfrac{1}{3}$ and $\sin x = -2$

_C_ 19. If an isosceles triangle has a base of length 36 and its vertex angle measures 48°, find the perimeter of the triangle.

    a. 75.4   b. 116.9   c. 124.5   d. 192